Finite Mathematics (Math 10120) Sec 01, Spring 2014

Solution

March 23, 2014

1. 16 teams enter a knockout tournament. In the first round, the teams split into 8 pairs (so if the teams are A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, then one possible splitting is that A plays B, C plays D, E plays F, G plays H, I plays J, K plays L, M plays N and O plays P). In how many ways can the teams be split into 8 pairs, if the order of the teams within a pair doesn't matter, and the order that the 8 pairs are announced doesn't matter (so, for example, A plays B, C plays D, E plays F, G plays H, I plays J, K plays L, M plays N and O plays P is the same as G plays H, P plays O, E plays F, B plays A, K plays L, I plays J, M plays N and C plays D)?

   Solution: Many people gave the answer $C(16,2)$; but this is just the number of ways of choosing **one pair** from among the 16 to play each other, and the question asks how many ways to *completely split up all 16 teams* into 8 pairs. Since we care neither about order of the pairs, or order within each pair, the answer is

$$\frac{\binom{16}{2,2,2,2,2,2,2,2}}{8!} = \frac{16!}{8!2!2!2!2!2!2!2!2!} = 2,027,025.$$

2. Suppose instead that in each pair, one team is to be designated as the home team, so the order of the teams within a pair *does* matter (A versus B is different from B versus A). Now how many ways can the teams be split into 8 pairs? (Again, the order that the pairs are announced doesn't matter, just the order *within* each pair).

   Solution: We multiply the answer from the previous part by 2^8, since for each of the 8 pairs, on after the other, we to choose the home team from among two possibilities. So the answer is

$$2,027,025 \times 2^8 = 518,918,400.$$